Sovereign Debt Issuance under Fiscal Budget Uncertainty and Market Frictions *

Alexander Eisl † Christian Ochs ‡ Stefan Pichler §

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PRELIMINARY AND INCOMPLETE

Abstract

We present a multi-period bond issuance model of a sovereign debt management office. Sovereign liquidity needs are assumed to be only predictable with error. In a framework of costly prediction errors, stochastic variation in the term structure of debt expenses and transaction costs, we comment on the optimal auction frequency determined by maturity allocation and issuance volume. Our model is consistent with an economic discussion on welfare effects of debt and contributes to finance literature by presenting the minimum cost-of-debt issuance strategy with respect to market frictions. Thus, we fill an existing gap between macroeconomic welfare maximization and cost minimization driven by accounting considerations.

Keywords: sovereign debt management, costs of debt, bond issuance

JEL classification: E61, G12, H63

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†WU (Vienna University of Economics and Business), Department of Finance, Accounting and Statistics, Welthandelsplatz 1, 1020 Vienna, Austria; *email:* alexander.eisl@wu.ac.at

‡WU (Vienna University of Economics and Business), Department of Finance, Accounting and Statistics, Welthandelsplatz 1, 1020 Vienna, Austria; *email:* christian.ochs@wu.ac.at

§WU (Vienna University of Economics and Business), Department of Finance, Accounting and Statistics, Welthandelsplatz 1, 1020 Vienna, Austria; *email:* stefan.pichler@wu.ac.at

1 Introduction

Government debt management has become increasingly important due to extensive requirements on public spending and soaring sovereign debt levels. Evidently, a sound basis for sovereign debt management plays an important role in supporting the government to meet its obligations and is key to fiscal and economic sustainability. In most countries, sovereign debt management is delegated to separate agencies called public debt management offices (DMOs). Despite operating in the country's name and account, day-to-day operations are carried out organizationally independent from the government. In this paper, we contribute to the sovereign debt managers problem by presenting a framework of optimal debt issuances under fiscal budget uncertainty in a market with frictions.

According to most DMO charters, the main target of those institutions is to minimize costs of public debt financing while avoiding excessive risk, i.e. as an indirect reference to the ESA95 manual on government debt (Office for Official Publications of the European Communities, 2002) and commitment to risk aversion. Despite this formal commitment, though, DMO objectives are very general in nature. Ideally, the government's debt management strategy should maximize welfare. The problem of optimal debt management therefore has many dimensions apart from institutionally driven cost-risk considerations, i.e., it refers to a broader, welfare based definition of costs and risk on a very long time horizon.

For example, as sovereign yield curves are widely used as a proxy for the risk free term structure, its replicability and the provision of liquidity services may play an important role for the economy. Furthermore, economic stability and time consistency of political decision making may be influenced by the maturity structure of debt. The public costs of ignoring these aspects of debt management may be considerable. From the debt managers perspective, the optimal strategy, considering all assets and liabilities, is hence the set of least distortive actions to the economy that is funding the government's operations. Cash management, in this respect, is an inherent task of DMOs due to potential time discrepancies between funding flows and spending requirements. In general, DMOs seem to be well aware of these duties.

However, even on a theoretical exercise, it is not always clear which welfare effects are dominating the multi-dimensional problem setting. There are many frictions that make it difficult to assess the potential factors that should be considered. In early macroeconomic models, the welfare optimizing issuance strategy followed the approach of insuring the fiscal budget against economic shocks. These models give rise to extreme fluctuations in the maturity structure of bonds, though, and are thus costly to implement in practice. Moreover, different economic shocks have very heterogenous effects on the budget. It turns out to be a difficult task to estimate the merits and detriments of such a strategy.

For welfare optimal debt management, a holistic view on the economy is absolutely required. In this paper, however, we do not attempt to solve this challenging task but contribute on an additional component of the optimal strategy. For this purpose, we analyze the debt managers issuance problem. In particular, we take a financial perspective on the economic trade-offs between management decisions in a market with frictions. We focus on empirically as well as institutionally relevant facts for our analysis. Thus, we elaborate on the following questions: i) how much cash holding is optimal? ii) which maturity composition is optimal under which conditions? iii) how should the optimal frequency of debt issues look like? In our view, this analysis fills an existing gap between macroeconomic theory and financial decisions taken by debt managers in markets with frictions and uncertainty.

We draw a number of parallels to debt management in corporations, but consider the sovereign debt management problem peculiar for several distinctive features. First, we argue that governments financing decisions have an impact on welfare, i.e. the optimal allocation between taxes and debt is fundamentally different from corporates. While taxation is mostly considered distortive and tax rates are therefore often rigid, governments have distinctive access to debt markets. For the same reason, we see different constraints and objectives in carrying out debt issuances. Second, sovereign debt issuances are conducted on a very frequent basis compared to corporates. As a result, governments face economically important higher order effects on their issuances, i.e. emerging patterns in the redemption schedule and accumulating interest payments have a significant impact on future liquidity needs and financing costs.

For a similarity with corporations, we argue that yield curves are in fact not fully endogenous to the government. Even though the government may frequently be the largest debtor in credit markets, it is not the only player to influence yields. On the one hand, independent central banks control the yield curve to a large extent in the short to medium run. On the other hand, sovereign debt is issued in increasingly competitive markets. First, debt of a particular state may be rivaled by other government's issues in the same currency (see Coppola et al. (2012)). Second, corporations also influence the yield curve and put it even more under control of capital markets (see Greenwood et al. (2010, 2015)). We therefore view sovereign interest expenses as being comprised of two components. In the first component, yields are exogenously determined by the secondary market. The other channel that affects yields is through the primary market and stems from endogenous market frictions and incompleteness, e.g. due to primary dealers' inventory risk (see Fleming and Rosenberg (2007)), funding constraints (see Brunnermeier and Pedersen (2008)), search- or replication costs. In particular, DMOs steer the timing, allocation and frequency of debt issues and thereby influence costs of friction.

We structure our paper as follows: Section 2 provides a survey of related literature and places our model in the relevant research context. In Section 3 we develop a framework for our model in a stepwise procedure, providing intuition of results at every stage of the process. Furthermore, we present the implications of our model and a comparative static analysis in Section 4. Finally, we conclude our findings in Section 5 and motivate further research.

2 Discussion of Related Literature

From a technical perspective, our research mostly relates to classical inventory management problems. In a sovereign debt market with frictions and uncertainty, we derive a simple framework of debt issuance that turns out to be a special case of the work of Van Zyl (1964); Nahmias and Pierskalla (1964). We extend their results and relate them to economic considerations on sovereign debt management. In our framework, costs of inventory depletion are mostly equivalent to the frictional costs associated with a shortfall on liquidity. Moreover, since a distinctive feature of debt instruments is their maturity, bonds are comparable to perishable goods with a fixed life-time. Thus, optimal ordering for inventory is in spirit similar to the optimal issuance strategy of governments with a stochastic demand for funding. By contrast to inventory management, though, the problem is complicated by deciding over the life-time of products, i.e., the maturity choice of sovereign bonds. In absence of the nice properties of the model from Van Zyl (1964); Nahmias and Pierskalla (1964), i.e. non-convexity in the maturity choice, we contribute by presenting a mixed-integer stochastic dynamic decision problem.

In addition, we relate our paper to a growing field of sovereign debt management literature filling a niche between macroeconomic welfare theory and institutionally driven decision making. While we take a partial view on only one component of government debt management, we draw the connection to the welfare effects of financial decision making. Since the early debate of Barro (1974) on welfare effects of government debt, sovereign debt management has departed from a pure interest expense driven approach to incorporate macroeconomic theory. The theory that positive welfare effects from providing smooth taxation dominate any additional interest expenses (Barro, 1979) has given rise to an asset liability management approach to debt management. This discussion has been formalized by Missale (1997) to a theory of optimal taxation. Even though tax smoothing may be discussed controversially as sole objective of debt management, the approach is supported by providing fiscal insurance to maintain political and economic stability of a country (Missale, 1999).

While Lucas and Stokey (1983) first showed that perfect fiscal budget insurance may be achieved by state contingent debt in a complete market, their work has been generalized to incomplete markets by Bohn (1988); Angeletos (2002). However, Buera and Nicolini (2004); Nosbusch (2008); Lustig et al. (2008) illustrate that those approaches require extreme positions in the maturity structure of debt which are not robust to different types of macroeconomic shocks. In a model that incorporates costs of misperception of the economy (Faraglia et al., 2010) or commitment costs (Debortoli et al., 2014), though, a portfolio that is balanced over each maturity is considered more robust and welfare efficient.

In our framework, we take perspective of a organizationally independent DMO deciding upon its auction schedule. For this purpose, we apply a rather narrow concept of debt expenses. However, we strongly promote that a broader cost concept must be considered (Blommestein and Hubig, 2012a,b) in order to comply with welfare optimal debt management. We refer to Blommestein and Hubig (2012b); Bernaschi et al. (2009) for a discussion of the relation between cost efficient debt management and alignment with welfare objectives. In particular, we rely on the assumption that yield curves are exogenous to the DMO. In this artificial setup, we analyze the impact of the optimal refinancing strategy on total debt expenses and attempt to reduce the gap from expense driven decision making to economic welfare theory. In our model, the source of welfare costs are market frictions in response to uncertainty in fiscal planning.

The preceding discussion indicates that optimality of applied debt management is difficult to assess. Apart from a focus on pure interest expenses, debt managers may pursue various objectives (Missale, 1999) that shall ultimately aim to maximize welfare. Consequently, measurements solely based on costs of debt may be biased and cannot be conducted to assess a debt managers performance. For example, Eisl et al. (2014) find significant market timing and maturity allocation effects but also significant underperformance with respect to a balanced strategy. They relate their result to objectives beyond the scope of interest expense minimization. On the other hand, Faraglia et al. (2008) only find a small impact of debt managers strategies on the objective fiscal insurance despite remarkable differences in the maturity composition across countries. Their findings confirm the extreme positions required to hedge the fiscal budget found by Buera and Nicolini (2004) and motivates that debt managers indeed control interest expenses apart from a pure hedging strategy.

While there are models applied at DMOs that simulate presumably optimal debt strategies with multiple objectives to support decision making (Tesouro Nacional, 2011;

Alves et al., 2008; Giavazzi and Missale, 2004; Melecky and Melecky, 2014), they are mostly tailored to practitioners needs and lack theoretic foundations. In addition, we do not comply with simulation based decision making that does not account for the stochastic nature of the decision problem. Instead, we focus on multi-stage stochastic dynamic programming (Date et al., 2011). We thrive to extend the scope of optimal debt management by analyzing the optimal strategy with respect to budget uncertainty in a framework that explicitly accounts for flexible future decision making.

3 A model of debt issuance strategies

We develop our model from specific to general, carefully building up intuition of results. We presume the following setup. A government's DMO has to finance any budget deficit in advance of upcoming periods to cover obligations duly. Debt may be raised via costly bond auctions to selected primary dealers at preset issuance dates. As observed empirically, the DMO announces information about the issued bonds some time prior to the auction. However, at the time of issuance the upcoming primary deficit is not known with certainty but is exposed to prediction errors and economic shocks.

We assume that primary dealers must be compensated for the costs incurred by an increase of the issued volume. In particular, any additionally issued amount must pay a spread in addition to planned bond issuances. In our model, this spread is the only source of variation in welfare and drives debt expenses of the DMO. Hence, the DMO must refer to cash management in order to insure against shocks to the fiscal budget. We offer two rationales for the required compensation of primary dealers. First, auction details are announced in advance of each auction in concordance with dealers expectations and economic demand. As primary dealers rely on this information, we assume that the required compensation are adjustment costs[1] of the auction calendar.

[1] These frictional costs could e.g. simply be of administrative nature or costs of excess inventory risk. For example, institutional constraints may call for hedging or rebalancing the portfolio. Further, we are concerned about dealers' funding constraints and inventory risk. See e.g. Fleming and Rosenberg (2007); Brunnermeier and Pedersen (2008) for a reference of frictions we consider.

As we will explain, any changes of the current auction may also alter the future auction schedule. Second, we could consider that the DMO falls short on funds in between auction dates. As a result, the DMO would need an inter-temporal loan to cover expenses duly. Of course, such a loan on short notice would be more expensive than funding that is well planned ahead. However, the results would qualitatively be left unchanged for either interpretation. We generally assume that anticipated issuances are cheaper than funding in reaction to a bad surprise.

3.1 Sequential Issuances

In a first step, we consider a simplified model from which much intuition can be derived. We impose that bonds are issued sequentially, i.e., one bond is issued at a time and is followed by a new issuance at redemption. In this simple model, the debt manager is risk neutral.

3.1.1 Optimal Issuance Size

In order to facilitate intuition, we assume that only one-period maturity bonds are available. The total amount of issuance I_t^{t+1} at time $t \in \{0, 1, \ldots, T-1\}$ finances the budget deficit B_t^{t+1} for one-period $[t, t+1)$. It comprises redemptions R_t, current debt expenses K_t and a capital buffer b_t^{t+1} as seen in Equation (1). Debt must be repaid in the next period in addition to further charges on interest k_t. However, stochastic financing needs may exceed the available funding by $(B_t^{t+1} - b_t^{t+1})^+$, which becomes known after issuance. A spread is charged on top of usual interest, i.e. $(k_t + s_t)$, to cover the mismatch duly. In each period, debt expenses are accrued according to Equation (2):

$$I_t^{t+1} = b_t^{t+1} + R_t + K_t \tag{1}$$

$$K_{t+1} = k_t I_t^{t+1} + (k_t + s_t)(B_t^{t+1} - b_t^{t+1})^+ \tag{2}$$

By time separability properties we may analyze debt expenses over one single period

$[t, t+1)$ to obtain the optimal cash buffer. In particular, the optimal capital buffer over one period is time consistent in the sense that it also minimizes total debt expenses over the whole time horizon if the policy is applied in each single period. See Equation (3) for the minimization problem over one period:

$$\min_{b_t^{t+1}} E[K] \text{ , where}$$

$$E[K] = k_t I_t^{t+1} + \int_b^\infty (k_t + s_t)(B_t^{t+1} - b_t^{t+1}) f_B(B_t^{t+1}) dB_t^{t+1} \tag{3}$$

The problem solves for the inverse cumulative distribution, i.e., the quantile function F_B^{-1} of the budget deficit:

$$b_t^{t+1} = F_B^{-1}\left(1 - E\left[\frac{k_t}{k_t + s_t}\right]\right) \tag{4}$$

The interpretation of result (4) is intuitive. As penalty costs s_t on on unanticipated budget deficits increase, more debt would be issued to avoid these states of the world. On the other hand, this increases costs of debt by the amount of cash holdings. Our result thus resembles the trade-off between issuing excess amounts of debt versus raising too little. As an illustration let us consider $s_t = 0$, i.e. there are no penalty costs on inter-temporal debt issuances. We would then intuitively conclude that it is optimal to issue debt just-in-time when obligations are due, i.e. $b_t^{t+1} = 0$ and roll-over known quantities such as redemptions and debt expenses. This guess holds true by budget deficits having only positive support and $b_t^{t+1} = F_B^{-1}(0) = 0$. See Figure 1 for a graphical illustration.

The above result, however, relies on the independence of the spread, coupon and budget deficit. Since budget deficits, coupons and spreads stem from the same state of the world, we generally assume a dependence structure. By contrast to the previous result, an analytical solution cannot be obtained in this case. Instead, we obtain the following formulation in Equation (8):

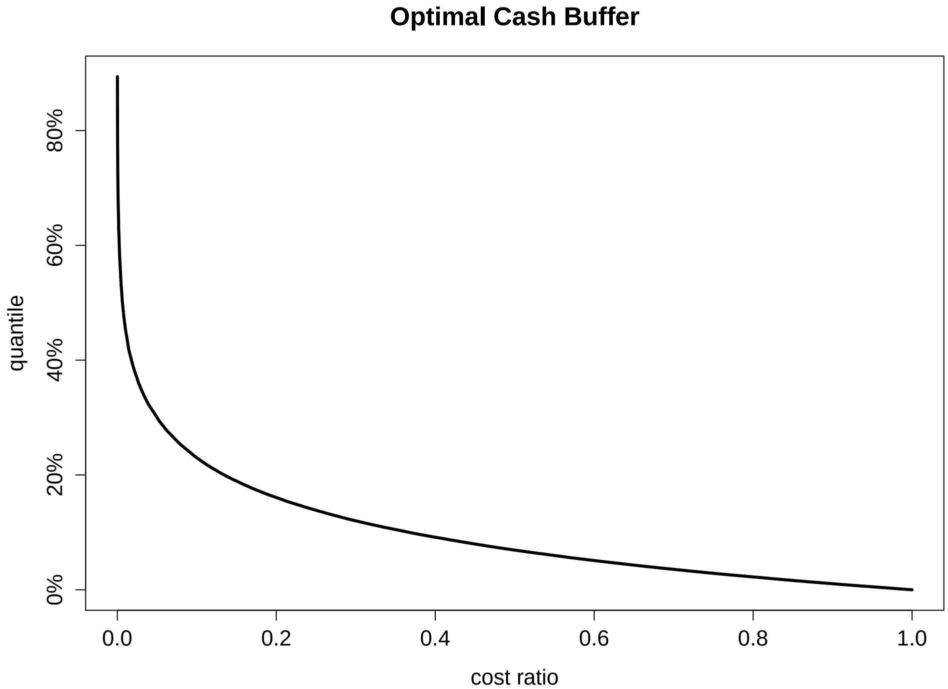

Figure 1: Graphical illustration of the optimality criterion stated in Equation (4). We plot the relationship between the optimal capital buffer, i.e. the quantile $F_B^{-1}(1 - E\left[\frac{k_t}{k_t+s_t}\right])$ of the budget deficit distribution, and the cost ratio $\frac{k_t}{k_t+s_t}$, i.e. costs of planned issues over costs of inter-temporal financing needs. For a Gamma distributed budget deficit, the capital buffer increases progressively with a decreasing cost ratio $\frac{k_t}{k_t+s_t}$. As spreads become more expensive, more debt would be issued in order to avoid penalty costs. For a ratio of unity, debt would be issued just-in-time as payments are due.

$$E[k_t \mathbb{1}_{\{B_t^{t+1} \leq b_t^{t+1}\}}] = E[s_t \mathbb{1}_{\{B_t^{t+1} > b_t^{t+1}\}}] \tag{5}$$

Thus, we come to the same intuitive result that expected marginal costs of issuing excessive debt must equal the expected marginal penalty costs on the inter-temporal loan. A numerical solution is easily obtained by an iterative procedure and allows e.g. endogenous spreads by the same equation.

3.1.2 Optimal Maturity Choice

For the choice of the optimal maturity, we generalize our model and allow to pick one from multiple available maturity dates m_{t_i} at each auction date t_i. For this purpose, we consider a term structure of coupon rates $k_{t_i}^{m_{t_i}}$. Bond issuances are again conducted sequentially.

$$I_{t_i}^{m_{t_i}} = b_{t_i}^{m_{t_i}} + R_{t_i} + K_{t_i} \tag{6}$$

$$K_{t_{i+1}} = k_{t_i}^{m_{t_i}} I_{t_i}^{m_{t_i}} + (k_{t_i}^{m_{t_i}} + s_{t_i}^{m_{t_i}}) \left(\sum_{u=m_{t_{i-1}}}^{t_i} B_u^{m_{t_i}} - b_{t_i}^{m_{t_i}} \right)^+ \tag{7}$$

The auction calendar is determined by the choice of the maturity, i.e. at each redemption date $t_i \in \tau := \{\tau_i \in [0,T] | \forall i \in \mathbb{N} : \tau_{i+1} = m_{t_i}, \tau_0 = 0 \text{ and } m_{t_i} \in \mu\}$ one bond with maturity date $m_{t_i} \in \mu := \{\mu_{t_i} \in [1,T] | \forall i \in \mathbb{N}, \forall j < k \in \tau : \mu_j < \mu_k, t_i \in \tau\}$ is issued. Thus, the number of auctions can be steered and is given by $\text{argmax}_i t_i$, where the maximum is attained at the last possible date $\max_i t_i = T$. Over the period $[t_i, m_{t_i})$ the optimal buffer is given by Equation (8):

$$b_{t_i}^{m_{t_i}} = (m_{t_i} - t_i) \cdot F_B^{-1} \left(1 - E \left[\frac{k_{t_i}^{m_{t_i}}}{k_{t_i}^{m_{t_i}} + s_{t_i}^{m_{t_i}}} \right] \right) \tag{8}$$

Total costs $\sum_{t_i \in \tau} K(m_{t_i})$ can thus be written as a function of the maturity choice in each period, where $K(m_{t_i}) := K_{t_{i+1}}$. Given a certain maturity, the optimal capital buffer is fixed. There is no advantage in altering the issued volume for a given maturity choice of debt. Intuitively, no cash can be transferred to subsequent periods since debt is issued sequentially, i.e., all outstanding debt is redeemed at the end of the period.

We may further simplify the previous statement (8) if we assume that the term structure of the spread exhibits the same functional form as the coupon itself. That is, expected penalty costs are proportionally invariant under the maturity horizon, e.g. a loan costs a multiple of a regular issue. Then, the optimal cash buffer is obtained by scaling up the one period solution:

$$b_{t_i}^{m_{t_i}} = (m_{t_i} - t_i) \cdot b_{t_i}^{t_i+1} \tag{9}$$

By contrast, the decision problem is generally not convex in the maturity choice and expectations about the term structure of interest rates. Given the optimal cash buffer, a change in the maturity may turn out either way. In particular, a timing ability of stochastic interest rates may allow refinancing at favourable terms in the future. Furthermore, any maturity decision will induce future liabilities of higher order, i.e., it will influence the timing of refinancing decisions itself. For this reason, the maturity choice is neither convex nor time consistent. The optimal maturity policy is spanned by the entire decision vector, i.e., the whole time horizon must be considered to derive the optimal maturity.

At date, a maturity M_0 is optimal for all maturity decisions and auction dates if $\forall m_u \in \mu_m, M_v \in \mu_M \exists M_0 \in \mu_M$:

$$E\left[K(M_0) + \sum_{u_i \in \tau_u} K(M_u)\right] \leq E\left[K(m_0) + \sum_{v_i \in \tau_v} K(m_v)\right] \tag{10}$$

Hence, we are in search of the optimal partition $M_0, M_v \in \mu_v$ of the intervall $[0, T]$. That is, one considers all future decisions to be taken optimally and compares the marginal impact of the maturity choice today. Thus, future redemption and refinancing dates are chosen ex-ante. Due to the stochastic refinancing costs, all future decisions at the chosen redemption dates have to be taken into consideration. Future decisions are adjusted dynamically by the same rational, i.e., the optimal policy responds to prevalent market conditions. By accounting for all future actions and consequences, this is approach is in line with the principle of optimality.

We use statement (10) for an analysis of the optimal maturity decision that is consistent with the expectation hypothesis. In the simplest case, the term structure is flat. Under this assumption, the maturity choice is considered irrelevant for a timing of issues since there are constant expectations about future interest rates. Debt is only raised for one period to minimize total debt expenses. However, yield curves are empirically increasing on average. Considering a normal yield curve, future interest rates are expected to rise. With debt expenses inreasing in the issued amount as well as in the coupon with maturity, debt managers would simply roll-over debt with the lowest coupon and shortest maturity. For this reason, an opaque maturity choice is time-consistent since it minimizes expected total debt expenses over the whole time horizon. Hence, under the expectation hypothesis and a monotone increasing term structure of coupons a maturity M_0 is optimal for the total amount of total debt expenses $\sum_{t_i \in \tau} K(m_{t_i})$ if $\forall m_0 \in \mu_m \exists M_0 \in \mu_M$:

$$E\left[K(M_0)\right] \leq E\left[K(m_0)\right] \tag{11}$$

Under general term structures, however, the optimal partition must be derived according to Equation (10). As a rational, locking in a favourable coupon rate over time may come at the costs of a higher volume to be issued. Thus, there is a trade-off between the benefit of timing issues by maturity and the costs incurred by a change in volume. In addition, redemptions have to be refinanced at an expected future coupon at the selected maturity dates. The debt manager cannot simply choose a combination of volume and coupon that minimizes the current costs and is optimal for total debt expenses. In order to capture all expectations about future yields, one must partition the whole planning horizon in advance. Ex-post, optimality of the maturity can obviously identified from a finite set of strategies, see e.g. Figure 2 for illustration.

Let us introduce fixed transaction costs for each auction conducted. Then, the general result in (11) is extended by the sum of transaction costs induced by the chosen maturity strategy, i.e., the frequency of auctions. Putting it differently, transaction costs can be considered as a penalty term on each issue that has considerable weight on short maturities. On average, fixed costs will thus increase the chosen maturity at each auction date. In particular, for the case of monotone increasing term structures fixed set-up costs will result in a more coarse partition of the time interval with a natural average distance between auctions.

However, market frictions render the choice of the maturity a even more complex task. First, auction set-up costs lead to an average auction frequency that restricts the optimal maturity choice as discussed. To complicate matters, there may be volume sensitive institutional costs that require a deviation from the average optimal strategy according to observed stochastic deficits. For example, the average auction frequency may be disturbed by attempts to avoid redemption clustering in response to unexpected deficits. Second, debt managers are considered to have certain expectations about future governance and debt expenses that offer opportunities to time debt issuances. Hence, by contrast to the expectation hypothesis world, an opaque strategy is not consistent with long term minimization of debt expenses. Lastly, debt managers are free to decide upon their decisions periodically and do not have to commit to sequential issuances. This eventually makes debt expenses a function of maturity and the cash buffer. For this reason, we analyze the optimal debt management strategy in a more general setting of dynamic decision making.

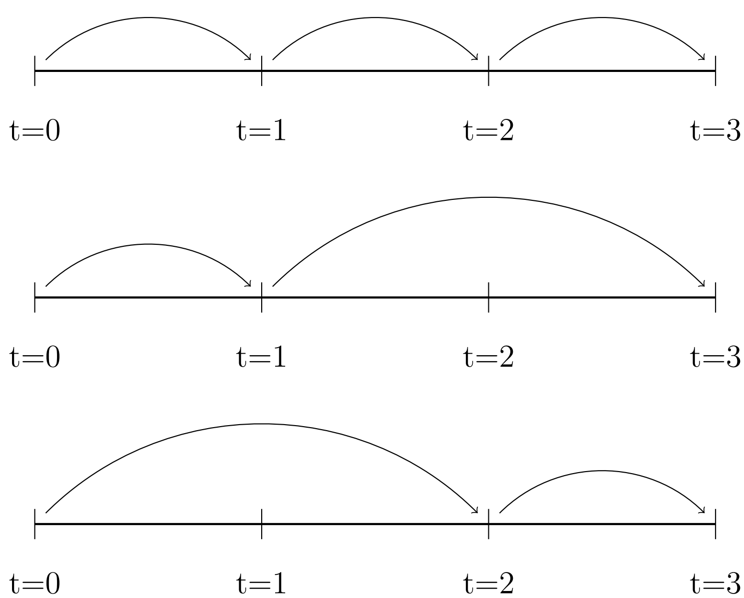

Figure 2: Illustration of the possible maturity strategies for the 3-period problem with sequential issuances. One strategy will turn out to be optimal ex-post. The optimal maturity is chosen ex-ante, though, while considering that all subsequent choices are taken optimally as well.

3.2 Dynamic Issuances

We generalize our model to dynamic bond issuances. That is, the debt manager is no longer restricted to sequential auctions and may choose from multiple available maturities. As a result, a number of variables and interactions among them become relevant for optimal decision making that did not play a role in the sequential setup. Most importantly, the maturity choice and choice of the issued volume must be jointly determined to minimize total debt expenses.

The DMO minimizes its costs of debt over the optimal maturity choice m_t and the capital buffer b_t. According to (13), costs of debt K_t consist of coupon payments $k_t^{m_t}$ on the total issuance amount $I_t^{m_t}$ for a given maturity date $m_t \in \{t+1, T\}$, plus coupon payments and spread charges $s_t^{m_t}$ on an inter-temporal loan $L_t^{m_t}$ in order to balance any deficit in excess of available funds. Only one maturity is issued at a time with all other maturity buckets $\neg m_t$ being set to zero $I_t^{\neg m_t} = L_t^{\neg m_t} = 0$. We will later consider the case of multiple maturities issued. In addition, there are transaction costs κ_t on each bond issuance. At the date of maturity m_t, redemptions R_t must be repaid in addition to debt expenses K_t. Cash holdings C_t are determined by Equation (17). The DMO problem can be formally stated as follows:

$$\min_{b,m} \ E\left[\sum_{t=0}^{T-1} K_{t+1}\right] \tag{12}$$

such that $\forall t$:

$$K_{t+1} = k_t^{m_t} I_t^{m_t} + (k_t^{m_t} + s_t^{m_t}) L_t^{m_t} + \kappa \tag{13}$$

$$I_t^{m_t} = b_t + R_t + K_t, \ I_t^{\neg m_t} = 0 \tag{14}$$

$$L_t^{m_t} = (B_t^{t+1} - C_t)^+, \ L_t^{\neg m_t} = 0 \tag{15}$$

$$R_t = \sum_{i=0}^{t} (I_i^t + L_i^t) \tag{16}$$

$$C_{t+1} = (C_t - B_t^{t+1})^+ + I_t^{m_t} - R_t - K_t \tag{17}$$

and where it holds $\forall t$ that:

$$B_t \sim F_B, k_t \sim F_k, s_t \sim F_s \tag{18}$$

$$m_t \in \{1, \ldots, T\} \tag{19}$$

$$b_t, K_t, R_t, C_t, I_t, L_t, \tau_t \in \mathbb{R}^+ \tag{20}$$

Evidently, total debt expenses $\sum_{t=0}^{T-1} K(m_t, b_t, C_t)$ are a function of maturity choice and cash buffer that depend on the current state and history. As such, decisions are time inconsistent, i.e., the whole time frame must be considered for an optimal policy. From a technical perspective, we face a highly non-linear, non-convex stochastic dynamic programming problem over a mixed-integer space. Due to the complexity of the model, we obtain our results by numerical optimization methods.

The decision model bears a number of trade-offs and interactions that emerge from the ability to carry cash over time. We discuss important mechanisms in the following and provide economic intuition of the non-linear trade-offs a debt manager is concerned with.

3.2.1 Trade-off decisions

Since the DMO is able to transfer cash C_t from one period to another by choosing maturity and issuance amount, transaction costs κ_t and the timing of coupons k_t become important determinants of total debt expenses. We discuss important trade-offs separately from each other.

The timing of coupons by issuing excess amounts of debt in one period to carry capital over to subsequent periods is eventually constrained by the maturity choice. As a result, the DMO is concerned with its cash management. There is a trade-off between taking benefit of a favourable coupon and the costs of carrying cash to subsequent periods. Since the DMO is able to dynamically adjust the issued volume in each period according to its cash reserves, the trade-off is more complex than simply locking in low interest rates like in the sequential setup. For example, the DMO may decide to finance only one period ahead but on a long maturity horizon to steer future redemptions and take advantage of the term structure. In addition, excessive debt could be issued in order to time future coupons and pre-issue for subsequent periods. Moreover, issuing debt today induces future liabilities via redemptions. Since DMOs issue debt on a regular and frequent schedule, there are substantial higher order effects that must be considered in timing coupons. Evidently, the cash buffer and the maturity must be jointy determined according to the cash state variable to obtain the optimal policy.

However, redemptions themselves could also be timed by taking out debt early to repay them in the future. Consider a fixed time horizon $[t, T)$ in which debt matures at some point $t + d$. The debt manager is concerned whether to pre-finance redemptions over the period $T - t$ or to wait and finance the period $T - d$. Holding everything else equal, the DMO would finance redemptions in advance, if condition (21) is satisfied:

$$E[k_{t+d}^{T-d}] \geq \frac{k_t^T}{(1 + dk_t^d)(1 - \frac{t}{T})} \tag{21}$$

That is, we obtain a function of the distance to redemption and the time horizon. The benefit of pre-financing at a certain interest rate is discounted by the costs of financing a longer period $[t, T)$ as well as the advantage to finance the period at k_t^d. Hence, the relative benefit must be considered. Despite this illustration, the optimal issuance amount must be determined jointly with the maturity choice taking all possible effects into consideration.

If transaction costs are considered fixed, there is a natural trade-off between the number of issues and the expected costs of carrying cash. Assuming transaction costs that are increasing in the auctioned volume, there may be an additional benefit in smoothing issued volume over time. The resulting costs and benefits may off-set incentives to time coupons. We generally consider transaction costs to be an affine function of the volume, i.e. a fixed amount plus proportional costs of the issued amount.

In this setup, we only consider single maturity auctions. However, we extend the model to multiple maturity auctions, i.e., a portfolio optimization approach. Indeed, one could argue that there are diversification benefits in response to macroeconomic shocks that affect the yield curve in different ways. In our view, though, diversification of the debt portfolio is achieved over time rather than at one single issuance. Since bonds may be issued with maturities longer than one period and dont have to be redeemed every period, it is possible to have several maturity buckets outstanding even if only one bond is issued at a time. Empirically, auctions of very few maturities are observed. We reason that transaction costs of additionally auctioned bonds are likely to outweigh the instantaneous diversification benefit. In addition, there is an actual time diversification benefit that arises by avoiding redemption clustering and timing cheap or excessive coupons at new issuances. However, under prevalent roll-over risk or present macroeconomic stress events there may be extra value added in the issuance of several maturities at once. We discuss these thoughts in the comparative static analysis in more detail.

In the light of the above, we consider the optimal issuance policy as the optimal frequency of bond auctions as a function of the cash buffer and maturity choice.

3.2.2 Picking the Optimal Strategy

In order to illustrate all trade-offs and interactions that are implied in our model, we consider a decision problem in 3 periods. Available maturities are one and two years. At the last stage, there is no budget deficit incurred and only redemptions are refinanced one more time. While the decision problem seems to be simple in three periods, it quickly grows out of hand.

The available maturity choices over time are illustrated in Figure 3. It is tempting to assume each of those maturity strategies separately and optimize the issuance amount. Ex-post, one would then pick the vector of maturities corresponding to the cheapest strategy.

However, this approach does not account for the stochastic nature of the decision problem. Decisions are taken dynamically as information is revealed over time. While one of the described strategies will be optimal ex-post, the risk of an adjustment of decisions is ignored if the strategy is considered in isolation. There may be implicit switching costs from one strategy to another. Hence, the principle of optimality would be violated. In order to solve our complex decision problem, we refer to multistage

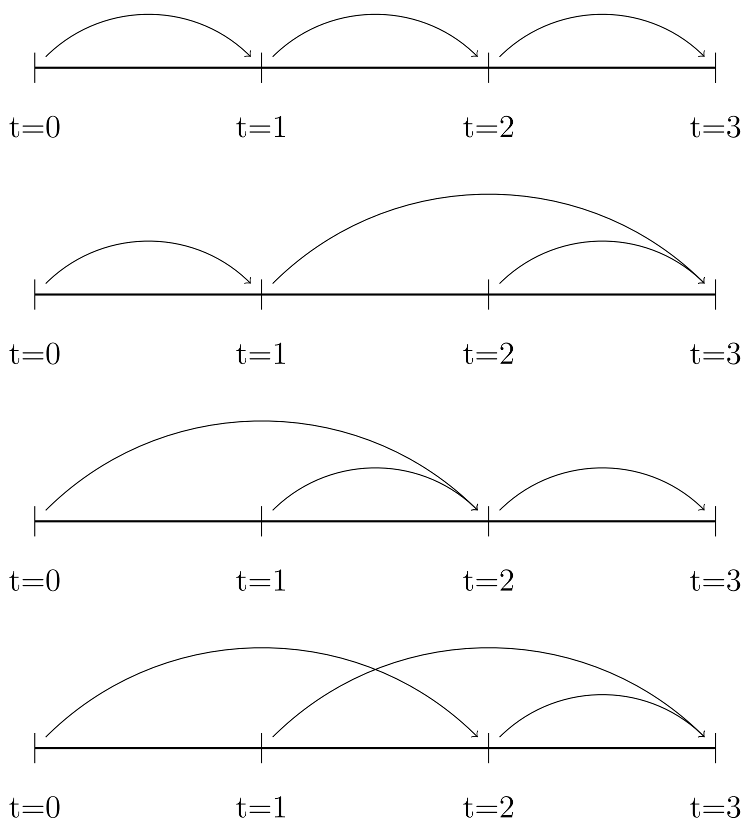

Figure 3: Possible maturity strategies for the 3-period problem. While one maturity strategy will be optimal ex-post, they must not be viewed in isolation. That is, the risk between strategies and switching costs must be considered ex-ante. Fixing the first decision, there may be a change of strategy between the top-two and between the bottom-two strategies. Strategies may be adjusted dynamically as time progresses.

stochastic programming with recourse.

4 A Comparative Static Analysis

The optimal maturity and issuance amount must be determined jointly taking into account dynamic decision making. For this reason, we choose a model of multi-stage stochastic recourse decisions that considers implicit switching costs and stochasticity of the optimization problem.

Being confronted with several future scenarios, the DMO is bound by its choice in each decision stage. As time progresses, more information is revealed and decisions can be adjusted accordingly. Recourse costs in our case can thus be seen debt expenses that result from a dynamic policy. That is, a flexible policy maintains feasibility in every state of the world and accounts for all scenarios of the underlying distribution. Choosing the least distortive strategy, however, comes at implicit cost that must be minimized, i.e., costs of adjusting decisions over time. Incorporating all future states of the world and all future decisions, the optimal strategy is chosen with respect to cost optimality. We operate on discrete scenario trees for this multi-stage stochastic programming problem as illustrated in Figure 4. Thus, the maturity choice and the cash buffer are jointly determined.

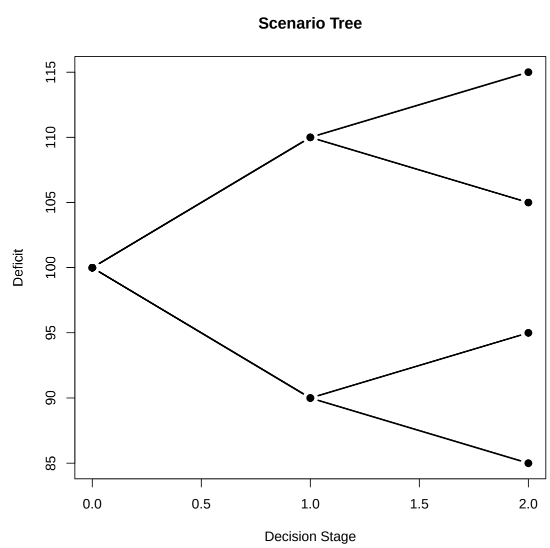

Figure 4: Graphical illustration of a scenario tree. Each node represents one possible state of the world. Information about budget deficits and interest rates is revealed over time. Decisions are set dynamically as time progresses. Thus, a change of strategies is accounted for and preserves the principle of optimality.

Scenario trees are generated according to the framework suggested by Hochreiter and Pflug (2014), i.e., backward distance minimization to obtain the optimal approximation of the underlying distribution. This flexible method allows us to consider simulated as well as real-world data in multiple dimensions. In particular, we are conducting a number of quality checks that include in-sample stability and out-of-sample stability of the problem solution, as well as robustness of the optimization method.

For future versions, we will extend our analysis to empirical data.

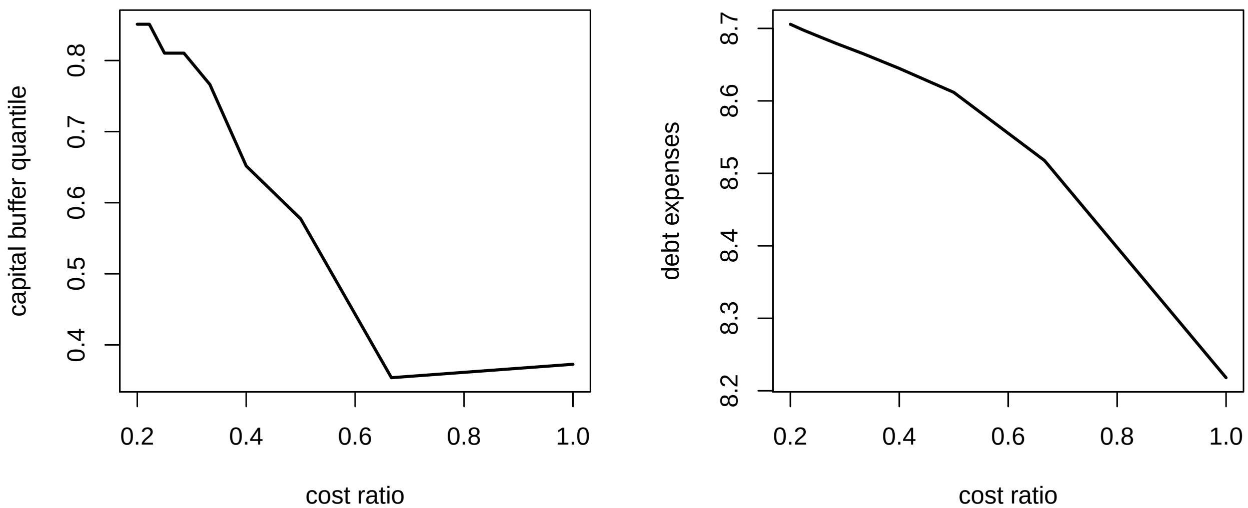

Figure 5: The left panel of this plot illustrates the optimal capital buffer $b_t^{m_t}$ as a function of the expected cost ratio $E\left[\frac{k}{k+s}\right]$. The buffer is expressed as the quantile of the underlying budget deficit. The right panel shows the relation of total debt expenses to the cost ratio.

A first[2] analysis of results illustrates the relation of the capital buffer as a function of the funding to penalty cost ratio as shown in Figure 5. As expected, asymmetric costs cause the DMO to hold cash over time. By contrast to our sequential setup, though, we observe capital buffers that undercut the expected value of the budget deficit. This is the result of imposing a heavily skewed gamma distribution of interest rates and budget deficits. In particular, the DMO faces frequent deficits that undercut the expected value and there is substantial probability mass on cheap refinancing opportunities in the future. On the other hand, there are significant costs for falling short of high budget deficits and facing expensive costs of refinancing. At this point, however, the debt manager has the flexibility to adjust the capital buffer in time to absorb unlikely but threatening shocks. This result contrasts the optimal policy of a sequential issuance

[2]Please note that this analysis is still work in progress and that results are subject to change. In particular, we are currently investigating the dynamics of our optimization system and will provide a more profound analysis soon.

setup.

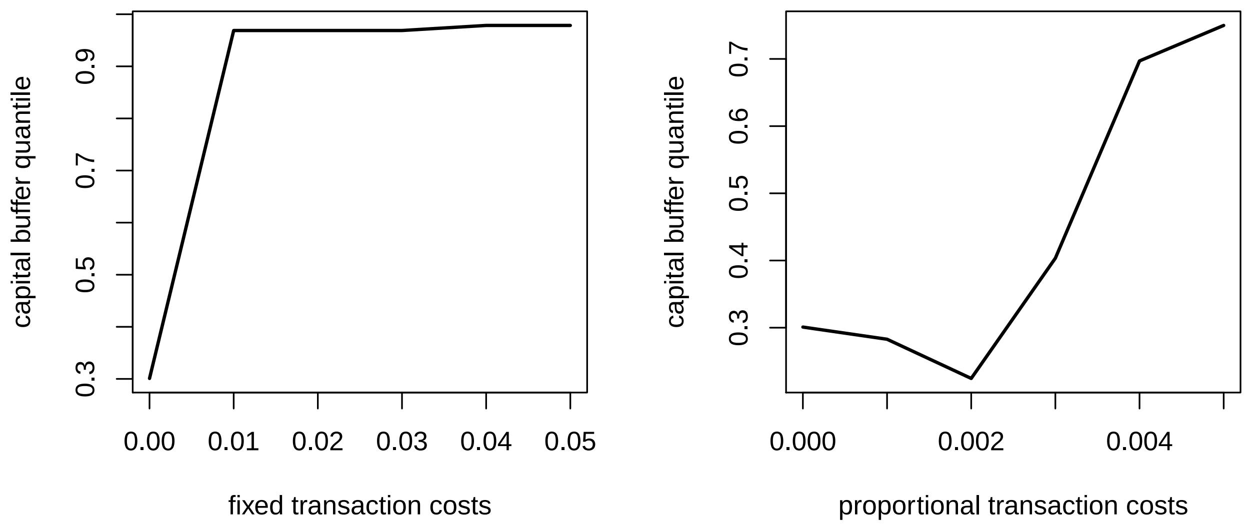

Figure 6: Fixed transaction costs are driving the capital buffer as seen in the left panel. The right panel exhibits the impact of proportional transaction costs on the capital buffer.

In addition, we find that the capital buffer is increasing in fixed transaction costs. If the DMO issues a larger capital buffer, this offers the option of leaving out auctions in subsequent periods. Hence, this result is expected and confirmed by empirical auction calendars. We also observe increasing capital buffers as a function of proportional, i.e., quadratic transaction costs. We attribute this finding to attempts of the debt manager to smooth redemptions over time and keep the overall issued volume low. In particular, we propose that the DMO would issue high capital buffers at certain maturities in order to avoid a clustering of redemptions. See Figure 6 for illustration.

These first results call for further research. An important factor of the optimal debt management strategy that needs to be investigated in detail is the influence of the maturity choice. This choice of the debt manager is of paramount importance in timing deficits and coupons as well as in dealing with transaction costs. Additionally, we will provide an empirical investigation of the DMO's auction schedule and derive institutional recommendations.

5 Conclusion

The recent sovereign debt crisis has sparked interest in the question of optimal debt management. However, it is difficult to derive an optimal debt management strategy as the objective function of a DMO is hard to define in practice. While prior research shows that it is optimal from a social welfare perspective to smooth tax rates, these models usually lead to very volatile strategies which are costly to implement when typical market frictions like transaction costs exist. On the contrary, these strategies may even undermine the objectives of welfare maximization in the presence of frictional costs. Other models have been developed to meet the requirements of practitioners but are very much accounting driven and mainly focus on reducing government deficits. It remains unclear if these models provide optimal results from a welfare perspective.

In this paper, we contribute to the sovereign debt literature by providing a multi-period model under fiscal budget uncertainty and realistic market frictions. Our model allows us to comment on several important aspects of DMOs issuance strategies while being consistent with a welfare maximizing objective.

First, we solve the DMO's cash holding problem in a sequential issuance setup. That is, one bond is outstanding at a time and a new auction is conducted at redemption. In a market setting with costly issues, a DMO wants to hold a sufficiently large cash buffer in order to balance the extra costs of holding cash with the risk of having to tap debt markets unexpectedly. Falling short of the planned budget triggers frictional costs that require increased compensation by the market. In a second step, we extend the model to allow a dynamic debt issuance strategy, i.e., a continous auction schedule that results in a portfolio of outstanding bonds. Due to the complexity of this model, we present a numerical procedure as an attempt to solve the debt managers problem. In this paper, we are investigating the dynamics of the optimization system and present a comparative static analysis.

Third, DMOs usually issue debt according to a pre-set auction calendar. Empirical evidence suggests that issuance strategies vary between countries. For example, Eisl et al. (2014) document cross-sectional differences in the frequencies of auctions. We attempt to analyze these empirical auction calendars with respect to the presented framework in a future version of our work. In particular, we are interested in how the optimal issuance frequency depends on transaction costs and how this interacts with optimal cash management of a DMO that is concerned with market frictions. While it seems almost impossible to provide a model that covers every possible market friction, we attempt to derive simple and intuitive policy implications which we believe can be implemented in practice.

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